Accuracy Arguments for Probabilism

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Degrees of Belief

1. Degrees of belief are **truth-value estimates.**
   - *Guesses.* "A miss is as good as a mile" standard of accuracy.
   - *Estimates.* "Closeness counts" standard of accuracy.*
2. **Probabilism:** Your degrees of belief should conform to the axioms of probability.
3. **The Norm of Gradational Accuracy:**

> An epistemically rational agent must evaluate [degrees of belief] on the basis of their gradational accuracy, and she must strive to hold a system of [degrees of belief] that, in her best judgment, is likely to have an overall level of gradational accuracy at least as high as that of any alternative system she might adopt.

*Estimates are evaluated on a gradational scale that assigns true beliefs higher degrees of accuracy the more strongly they are held, and false beliefs lower degrees of accuracy the more strongly they are held.

Joyce's Plan: (1) Give an account of what it is for degrees of belief to accurately represent the world. (2) Explain why having degrees of belief that conform to the axioms of probability contributes to the epistemic goal of accuracy.

The Formal Framework

- The agent has beliefs in some finite, ordered set of propositions $\hat{X} = \langle x_1, x_2, ..., x_n \rangle$.
- B is the family of all credence functions for propositions in $\hat{X}$. Elements of B are vectors $\hat{b} = \langle b_1, b_2, ..., b_n \rangle = \langle b(x_1), b(x_2), ..., b(x_n) \rangle$. We will assume that each $b_j \in [0, 1]$.
- $\hat{V} = \{\omega^1, \omega^2, ..., \omega^n\}$ is the subset of B that contains all consistent truth-value assignments to elements of $\hat{X}$. Each element of $\hat{V}$ is a binary sequence $\hat{\omega} = \langle \omega_1, \omega_2, ..., \omega_n \rangle$, where 0 is false and 1 is true.
- The set of all probability assignments over $\hat{X}$ is the convex hull $\hat{V}^+$ of $\hat{V}$.

Measures of Inaccuracy

A *measure of gradational accuracy* is a function from belief-states and worlds to real numbers $I(\hat{b}, \hat{\omega})$. Assume that I is continuous, that $I(\hat{b}, \hat{\omega}) \geq 0$, and that $I(\hat{b}, \hat{\omega}) = 0$, when $\hat{b} = \hat{\omega}$.

Examples: Measures of the form $I_p(\hat{b}, \hat{\omega}) = \sum_j \lambda_j |b_j - \omega_j|^p$

(Ab) The Absolute Value Measure.

$$I_p(\hat{b}, \hat{\omega}) = \textstyle\sum_j \frac{1}{n} |b_j - \omega_j|$$

(Br) The Brier Score.

$$I_p(\hat{b}, \hat{\omega}) = \textstyle\sum_j \frac{1}{n} |b_j - \omega_j|^2$$

(EJ) "Extreme Jamesian" Functions. (Concave to Flat)

$$1 \geq p > 0.$$

(Ja) "Jamesian" Functions. (Mildly Convex, not Symmetric)

$$2 > p > 1$$

(Cl) "Cliffordian" Functions. (Really Convex, not Symmetric)

$$p > 2$$

Constraints on Measures of Inaccuracy:

1. **Structure.** For each $\omega \in V$, $I(b,\omega)$ is a non-negative, continuous function of b that goes to infinity in the limit as $b(X)$ goes to infinity for any $X \in \Omega$

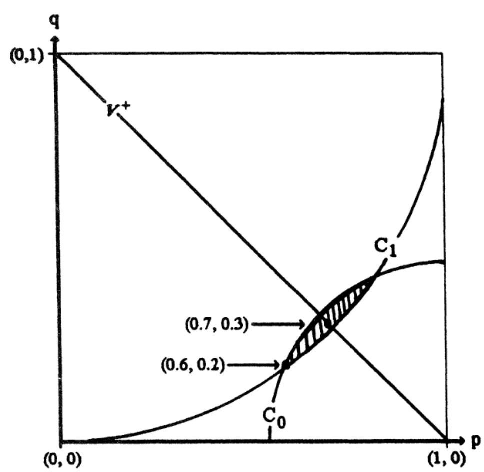
2. **Extensionality.** At each possible world ω, $I(b,\omega)$ is function of nothing other than the truth-values that ω assigns to propositions in Ω and the degrees of confidence that b assigns these propositions.

3. **Dominance.** If $b(Y) = b^*(Y)$ for every $Y \in \Omega$ other than X, then $I(b,\omega) > I(b^*,\omega)$ if and only if $|\omega(X) - b(X)| > |\omega(X) - b^*(X)|$.

4. **Normality.** If $|\omega(X) - b(X)| = |\omega^*(X) - b^*(X)|$ for all $X \in \Omega$, then $I(b,\omega) = I(b^*,\omega^*)$.

5. **Weak Convexity.** Let $m = (\frac{1}{2}b + \frac{1}{2}b^*)$ be the midpoint of the line segment between b and b^*. If $I(b,\omega) = I(b^*,\omega)$, then it will always be the case that $I(b,\omega) \geq I(m,\omega)$ with identity only when $b = b^*$.

6. **Symmetry.** If $I(b,\omega) = I(b^*,\omega)$, then for any $\lambda \in [0,1]$ one has $I(\lambda b + (1-\lambda)b^*,\omega) = I((1-\lambda)b + \lambda b^*,\omega)$



The Main Theorem. If gradational inaccuracy is measured by a function I that satisfies Structure, Extensionality, Dominance, Normality, Weak Convexity, and Symmetry, then for each $c^* \in B \setminus V^+$ there is a $c \in V^+$ such that $I(c^*,\omega) > I(c,\omega)$.

Every non-probabilistic belief-state is *accuracy dominated* by some probabilistic belief-state. So, if accuracy is a central epistemic goal, we have a reason to adopt degrees of belief that conform to the probability axioms.

Structure says that inaccuracy should be non-negative, that small changes in degrees of belief should not engender large changes in accuracy, and that inaccuracy should increase without limit as degrees of belief move further and further from the truth-values of the propositions believed.

Extensionality stipulates that the "facts" which a person's partial beliefs must "fit" are exhausted by the truth-values of the propositions believed, and that the only aspect of her opinions that matter is their strengths.

Dominance requires the accuracy of a system of degrees of belief to be an increasing function of the believer's degree of confidence in any truth and a decreasing function of her degree of confidence in any falsehood.

This principle really says two things. First, it lets us speak of the accuracy of each individual degree of belief taken *in isolation* from the belief system as a whole. Second, it says that the accuracy of $b(X)$ always increases as it approaches $\omega(X)$. Thus, moving one's degree of belief for X closer to X's truth-value improves accuracy *no matter what one's other degrees of belief might be.*

Normality says that differences among possible worlds that are not reflected in differences among truth-values of proposition that the agent believes should have no effect on the way in which accuracy is measured.

In the presence of the other conditions, this merely says that the standard of gradational accuracy must not vary with changes in the world's state that do not effect the truth-values of believed propositions.

Weak Convexity and **Symmetry** rule out inaccuracy measures, like the Absolute Value Measure, that (Joyce claims) "privilege" some "dimensions of credal space" over others.

Figure (above) is from *Bristol Summer School: Introduction to Epistemic Utility Theory* website: `https://sites.google.com/site/bristolsummerschool/`

The shaded region represents all the belief-states that *accuracy-dominate* the non-probabilistic belief-state $c^* = \langle 0.6, 0.2\rangle$.

The line marked V^+ represents all the probabilistic belief-states, including $c = \langle 0.7, 0.3\rangle$.

The curve C_0 represents all the belief-states b such that $I(c^*,p) = I(b,p)$, and the curve C_1 represents all the belief-states b' such that $I(c^*,q) = I(b',q)$, where $p = \langle 1,0\rangle$, and $q = \langle 0,1\rangle$.