Dutch Book Arguments for Probabilism

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Betting Interpretation of Degrees of Belief

What is your degree of belief in the proposition A?

$$\mathbf{B1} = \begin{cases} \$(1-p)\cdot S & \text{if } A \\ -\$p \cdot S & \text{o.w.} \end{cases} \qquad \mathbf{B2} = \begin{cases} \$p\cdot S & \text{if } \neg A \\ -\$(1-p) \cdot S & \text{o.w.} \end{cases}$$

Find the value of p such that you are *indifferent* between **B1** and **B2.**

You are indifferent between X and Y just in case you don't prefer either one to the other.

FAIR BETTING RATE: Call this value of p your *fair betting rate.*

Your *credence* in A is identified with your fair betting rate: $Cr(A) = p$.

A Dutch Book

Suppose that $Cr(A) = p$ and $Cr(\neg A) = q$, such that $p + q > 1$. These degrees of belief violate the axioms of probability.

If $p + q < 1$, then we'd look at the amount at which you'd be willing to *sell* the following two bets.

Consider the following two bets:

$$\mathbf{Bet}(A) = \begin{cases} \$S & \text{if } A \\ -\$0 & \text{o.w.} \end{cases} \qquad \mathbf{Bet}(\neg A) = \begin{cases} \$S & \text{if } \neg A \\ -\$0 & \text{o.w.} \end{cases}$$

Given your credences, you'd pay $\$S \cdot p$ for $\mathbf{Bet}(A)$ and $\$S \cdot q$ for $\mathbf{Bet}(\neg A)$. But paying these prices for both bets guarantees you a sure loss:

	A is true	$\neg A$ is true
Buy $\mathbf{Bet}(A)$	$S - S \cdot p$	$-S \cdot p$
Buy $\mathbf{Bet}(\neg A)$	$-S \cdot q$	$S - S \cdot q$
Total:	$S \cdot (1 - (p+q))$	$S \cdot (1 - (p+q))$

Because, by hypothesis, $p + q > 1$, the result of taking both these bets at their corresponding rates, i.e. $S(1 - (p+q))$, is negative. No matter what — regardless of whether A is true or $\neg A$ is true — you will lose money.

Dutch Book Arguments

Why should your degrees of belief conform the probability axioms? *Dutch Book Arguments* attempt to ground the irrationality of non-probabilistic degrees of belief by pointing to betting situations like the one above.

Pragmatic-Consistency Argument

P1 If your credence function is not a probability function, then you are hypothetically vulnerable to guaranteed loss.

P2 If you are hypothetically vulnerable to guaranteed loss, then you are irrational.

C If your credence function is not a probability function, then you are irrational.

Is **P1** true?

- *Operationalism about Degrees of Belief.* What it is for $Cr(A) = p$ just is for you to be indifferent between **B1** and **B2.**
- *Functionalism about Degrees of Belief.* What it is for $Cr(A) = p$ is partly constituted by preferences over bets that turn on A.

But need there be a tight metaphysical connection between your degrees of belief and your preferences over bets?

Is **P2** true?

- Is "hypothetical vulnerability" enough? I know that there are no cunning bookies around.

Why should I care about a guaranteed loss that I would face in a situation that I know I will never be in?

These kinds of Dutch Book Arguments locates the irrationality of your non-probabilistic degrees of belief in your *preferences*? Do these arguments, at best, show that you have a *pragmatic* reason to have probabilistic credences?

Howson Belief Argument

P1 If your credence function is not a probability function, then it is you believe that buying **Bet**(A) for $\$Sp$ is fair and you believe that buying **Bet**$(\neg A)$ for $\$Sq$ is fair.

P2 If you believe that buying X for price $\$x$ is fair and you believe that buying Y for price $\$y$ is fair, then you believe that buying $X\&Y$ for price $\$(x+y)$ is fair.

P3 You don't believe that buying **Bet**(A) & **Bet**$(\neg A)$ for a price of $\$S \cdot (p+q)$ is fair.

C If your credence function is not a probability function, then you have inconsistent beliefs.

Why think that having certain credences entails that you have the corresponding beliefs about bets?

Christensen Depragmatized Dutch Book Argument

P1 If it's rationally permissible that your credence function not be a probability function, then it's rational to believe that buying **Bet**(A) for $\$Sp$ is fair and it's rational to believe that buying **Bet**$(\neg A)$ for $\$Sq$ is fair.

P2 If it's rational to believe that buying X for price $\$x$ is fair and it's rational to believe that buying Y for price $\$y$ is fair, then it is rational to believe that buying $X\&Y$ for price $\$(x+y)$ is fair.

P3 It is not rational to believe that buying **Bet**(A) & **Bet**$(\neg A)$ for a price of $\$S \cdot (p+q)$ is fair.

C It is not rationally permissible for your credence function to be not a probability function.

Christensen uses the phrase "sanctions as fair." His argument differs from the previous one in that the connection between your degrees of belief and your evaluations of bets is not a *metaphysical connection* — rather, it is a *normative* connection. Having certain degrees of belief "sanction as fair" certain bets.