Completeness & Parity

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The Completeness Axiom

Another constraint on your preference is that they be *complete*:

Completeness For any X and Y, either $X \succ Y$, $Y \succ X$, or $X \approx Y$.

If your preferences fail to be complete, they cannot be represented with a utility-function. For any numbers, r_1 and r_2, either $r_1 > r_2$, $r_2 > r_1$, or $r_1 = r_2$.

Are you irrational if your preferences fail to be complete? Most people think "No" (the constraint is often assumed for mathematical convenience). Why might it be rational to have incomplete preferences?

> [W]e evaluate prospects on a variety of "scales" of goodness, and there is no reason, in general, to think that these can be amalgamated in any satisfactory way to yield a single unitary measure of value. Some goods (or ways of being good) are simply *incommensurable* with others.

James Joyce, *The Foundations of Causal Decision Theory*, Cambridge University Press. 1999. p. 99–101

Ruth Chang argues that we should recognize a fourth, *sui generis* value-relation: *Parity*.

Suppose you are facing a "hard choice" between two alternatives: (1) one alternative is better in some relevant respects; (2) the other alternative is better in other relevant respects; and yet (3) neither seems to be at least as good as the other in all relevant respects. Chang argues that, in these case, you regard the two alternatives as *on a par*.

Value Relation	Bias	Nonzero Magnitude?
X is better than Y	Yes (toward X)	Yes
Y is better than X	Yes (toward Y)	Yes
X and Y are equally good	No	No
X and Y are on a par	No	Yes

What distinguishes parity from indifference? The former is *insensitive to mild sweetening*:

Small Improvements Argument

P1 X is neither better nor worse than Y.

P2 X^+ is better than X.

P3 X^+ is neither better nor worse than Y.

C X and Y are not equally good.

The Puzzle of Opaque Sweetening

Suppose that regard A and B as on a par. A fair coin has been flipped. If it landed heads, then A was placed in the Larger box and B was placed in the Regular box. If it landed tails, then B was placed in the Larger box and A was placed in the Regular box. A dollar is added to the Larger box; nothing is added to the Regular box.

	Heads	Tails
L	$A^{+\$1}$	$B^{+\$1}$
R	B	A

You Ought to Take L

L's prospects are $\left\{\left\langle \frac{1}{2}, A^+ \right\rangle, \left\langle \frac{1}{2}, B^+ \right\rangle\right\}$. R's prospects are $\left\{\left\langle \frac{1}{2}, A \right\rangle, \left\langle \frac{1}{2}, B \right\rangle\right\}$.

- **Prospect Argument.** L has better *prospects* than R. You should evaluate your options solely in terms of their corresponding prospects. Therefore, you should take L.
- **Reasons Argument.** You have a reason to take L rather than R (you'll get a dollar). You have no reason to take R over L (everything that can be said in favor of taking R can equally well be said in favor of L). Rationality requires you to do what you have the most reason to do. Therefore, you should take L.

Prospectism: Consider the set of complete coherent extensions of your incomplete preferences. Associate with each complete ordering a utility-function. If an alternative maximizes expected utility with respect to *all* of these utility-functions, you are rationally required to take it.

Because $A^+ \succ A$ and $B^+ \succ B$, every utility-function in the set ranks A^+ ahead of A and B^+ ahead of B. Let u be an arbitrary utility-function from the set.

$$EU(L) = \frac{1}{2} \cdot u\left(A^+\right) + \frac{1}{2} \cdot u\left(B^+\right)$$
$$EU(R) = \frac{1}{2} \cdot u\left(A\right) + \frac{1}{2} \cdot u\left(B\right)$$

And so $EU(L) > EU(R)$ because $u\left(A^+\right) - u\left(A\right) > 0$ and $u\left(B^+\right) - u\left(B\right) > 0$. This holds for every utility-function. Therefore, L is ranked ahead of R with respect to every function in the set. And therefore, according to *Prospectism*, you ought to prefer L to R.

It's Permissible to Take Either

- **Dominance.** R never does worse than L (for each state S, you don't prefer $(L \wedge S)$ to $(R \wedge S)$). If an alternative never does worse than the others available, it's permissible to take it. Therefore, it's permissible to take either box.
- **Deference/Reflection.** Any fully-informed, rational person with all and only your preferences over outcomes will not prefer L to R. If any fully-informed, rational person with all and only your preferences over outcomes has an array of preferences over alternatives, it's permissible for you to adopt that array of preferences. Therefore, it's permissible for you to not prefer L over R; and so it's permissible to take either box.
- **Actual Value.** If you know that the actual value of an alternative doesn't exceed the actual value of the other, then it's permissible to take either. You know that L's actual value doesn't exceed R's actual value. Therefore, it's permissible to take either.

(What would a fully general decision theory that gets this result look like?)