Three Puzzles of Confirmation

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Bayesian Confirmation Theory

What is it for some evidence E to provide some confirmation for a hypothesis H?

Bayesian Confirmation Theory.

Evidence E *confirms* hypothesis H just in case

$$\Pr(H \mid E) > \Pr(H) \tag{1}$$

We are using 'confirms' in a technical sense to mean something like "E is evidence for H" or "E supports H," etc.

What does it take for some evidence to confirm the hypothesis?

From Bayes' Theorem:

$$\begin{aligned}\Pr(H \mid E) &= \frac{\Pr(E \mid H) \cdot \Pr(H)}{\Pr(E)} \\ &= \frac{\Pr(E \mid H)}{\Pr(E)} \cdot \Pr(H)\end{aligned}$$

So, E confirms H just in case:

$$\Pr(E \mid H) > \Pr(E)$$

This is because:

$$\begin{aligned}\Pr(H \mid E) &> \Pr(H) \\ \frac{\Pr(E \mid H)}{\Pr(E)} \cdot \Pr(H) &> \Pr(H) \\ \frac{\Pr(E \mid H)}{\Pr(E)} &> 1\end{aligned}$$

So, $\Pr(H \mid E) > \Pr(H)$ just in case $\Pr(E \mid H) > \Pr(E)$.

Consider the following principle. Is it true?

> Instance Principle: Observations of instances of a generalization *confirm* that generalization.

Generalizations have the form

All Fs are G.

Observing an instance of this generalization would be to learn of some particular F, a, that a is G.

Nelson Goodman's "New Riddle of Induction"

Consider the following properties:

$$x \text{ is } \textit{grue} \text{ iff}_{df} \begin{cases} x \text{ is green} & \text{if } x \text{ is observed before 2020} \\ x \text{ is blue} & \text{if } x \text{ is not observed before 2020} \end{cases}$$

$$x \text{ is } \textit{bleen} \text{ iff}_{df} \begin{cases} x \text{ is blue} & \text{if } x \text{ is observed before 2020} \\ x \text{ is green} & \text{if } x \text{ is not observed before 2020} \end{cases}$$

Suppose that you observe a green emerald. Does this confirm the hypothesis *"All emeralds are grue"*?

$$\Pr(E \mid H) > \Pr(E) \text{ ?}$$

$E = (\text{Emerald}(a) \wedge \text{Grue}(a))$.

$H =$ *All emeralds are grue.*

According to Bayesian Confirmation Theory, it appears that it does. But is this the right thing to say about this?

The Paradox of the Ravens

Consider the following hypothesis:

H = *All Fs are G.*

Hypothesis H is logically equivalent to hypothesis H^*

H^* = *All non-G things are non-Fs.*

If the INSTANCE PRINCIPLE is correct, then observing that some non-G thing is also a non-F, confirms the hypothesis H^*. But, because H and H^* are logically equivalent, such an observation also confirms hypothesis H.

Example. Observing that this white shoe is not a raven confirms the hypothesis that all ravens are black.

But that seems absurd! Indoor ornithology?

We can translate H into

For all things x, if x is an F, then x is a G.

And we can translate H^* into

For all things x, if x is not a G, then x is not an F.

And both statements can be understood as saying "for all things x, either x is not F, or x is G." They are logically equivalent.

Example. Consider the hypothesis:

Everyone in here drinking alcohol is over 21 years old.

And now imagine two different situations.

(1) You are in a crowded bar in which the majority of people are drinking.

(2) You are in a family restaurant in which only very few people are drinking.

Offhand, in (1), observing that someone under 21 is not drinking provides more confirmation for the hypothesis than observing that someone drinking is over 21; and in (2), observing that someone drinking is over 21 provides stronger confirmation than observing that someone under 21 is not drinking.

The "Problem" of Irrelevant Conjunctions

Carnap distinguished *incremental confirmation* (probability raising) from *absolute confirmation* (higher posterior probability).

TWO PRINCIPLES OF INDUCTIVE LOGIC

Special Consequence Condition (SC): If E confirms H, and H entails H^*, then E confirms H^*.

Converse Consequence Condition (CC): If E confirms H, and H^* entails H, then E confirms H^*.

Problem: These two principles entail that if E confirms something, then E confirms anything!

Proof of the Problem. Suppose that E confirms H. We will show that for any A, E then confirms A. From **(CC)**, E confirms $H \wedge A$. And $H \wedge A$ entails A, so, by **(SC)**, E confirms A. QED.

1. ***Counterexample to (SC)? Incremental Confirmation.*** Let K =*My pet is either a lizard or a dog.* Let E =*My pet has no hair.* Getting evidence E confirms "my pet is a hairless dog" but it does not confirm "My pet is a dog."

2. ***Counterexample to (SC)? Absolute Confirmation.*** This principle holds for absolute confirmation. If H entails H^*, then $\Pr(H^* \mid E) \geq \Pr(H \mid E)$. So, if $\Pr(H \mid E)$ is high, then $\Pr(H^* \mid E)$ must be high too.

The "Problem" of Irrelevant Conjunctions: Let $H^* = H \wedge J$, where J is something "irrelevant." According to Bayesian Confirmation Theory, any evidence E that confirms H, will also confirm H^*.
But that's weird! (Is it?)

Where J is "irrelevant" just so long as it is *probabilistically independent* of H, E, and $H \wedge E$.