

Amartya Sen's Liberal Paradox

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Collective Rationality and Social Choice Theory

We've focused on questions about *individual* rationality. Now we turn to questions of *social* or *collective* rationality—given that our interests conflict, what are *we* to do? Questions of this sort are studied in *social choice theory*.

Social Choice Framework. Let χ be the set of all possible social states (each one being a complete description of society including every individual's position within it).

For each individual, i , let \succ_i be individual i 's preference-ranking over the members of χ (we assume that each of these rankings are transitive and complete). The set consisting of every individual's preference-ranking, we will call a *preference profile*.

Let a *collective choice rule* be a function from preference profiles to one (and only one) social preference relation: \succ_s . We will require every \succ_s to generate a choice-function that specifies, for every subset of alternatives from χ , a subset of "maximal" options.

Sen's argument shows that there is a conflict between *liberty* and *efficiency*.

Impossibility of a Partial Liberal

Sen proves that the following constraints on a collective choice rule cannot be jointly satisfied.

PARETO: If everyone ranks X ahead of Y , then X must be socially preferred to Y .

$$\text{If, } \forall i, X \succ_i Y, \text{ then } X \succ_s Y.$$

LIBERTY: For each individual, i , there is at least one pair of alternatives, (X, Y) , such that i 's ranking of these alternatives is decisive (that is: if i prefers X to Y , then X must be socially preferred to Y ; and, if i prefers Y to X , then Y must be socially preferred to X).

$$\forall i, \exists X, Y, X \succ_i Y \text{ if and only if } X \succ_s Y.$$

UNRESTRICTED DOMAIN: Every logically possible set of individual orderings is included in the domain of the collective choice rule.

As we will soon see, there are various ways to understand this question. For example,

1. Which outcomes would an impartial, benevolent spectator prefer?
2. Which outcomes should our democratic institutions be designed to bring about?
3. Given each citizen's individual preference-ordering, what does "society" prefer? (Or, if this is different, what *should* it prefer?)

More generally, are we asking a *descriptive* question (e.g., "What does that group want?"), or an *evaluative* question (e.g., "Which outcome is best (overall, from an agent-neutral perspective)?"), or a *normative* question (e.g., "Which outcome should we try to bring about?"), or something else?

If we require \succ_s to be an ordering—transitive, complete, etc.—then we will call the rule a *social choice function*.

Famously (in his Nobel-prize-winning doctoral dissertation), Kenneth Arrow proved that there can be no social welfare function satisfying a small number of independently plausible constraints. This is known as *Arrow's Impossibility Theorem*.

The idea behind the **LIBERTY** constraint is that everyone should be granted a *recognized personal sphere* over which one has liberty—regarding such matters, one's preferences (and only one's) count in determining what ought to happen.

- o *Examples:* What color to paint your wall. Whether to sleep on your back or your side. Which books to read. Whether to pray. Etc.

Lady Chatterly's Lover

Suppose there are two individuals: *Prude* (p) and *Lewd* (l). The question concerns who, if anyone, should read the risqué *Lady Chatterly's Lover*.

There are four relevant alternatives:

| | p | l |
|--------------------------------|------------------------|-----|
| \emptyset : No one reads it. | $\langle 0, 0 \rangle$ | |
| P : Only Prude reads it. | $\langle 1, 0 \rangle$ | |
| L : Only Lewd reads it. | $\langle 0, 1 \rangle$ | |
| E : Everyone reads it. | $\langle 1, 1 \rangle$ | |

Liberal Idea: If the difference between two alternatives only legitimately affects individual i , then i 's preferences over those alternatives should be decisive.

- The choice between $\{\emptyset, P\}$ only affects Prude.
Because $\emptyset \succ_p P$, $\emptyset \succ_s P$.
- The choice between $\{E, L\}$ only affects Prude.
Because $L \succ_p E$, $L \succ_s E$.
- The choice between $\{\emptyset, L\}$ only affects Lewd.
Because $L \succ_l \emptyset$, $L \succ_s \emptyset$.
- The choice between $\{E, P\}$ only affects Lewd.
Because $E \succ_l P$, $E \succ_s P$.

Pareto: Both Prude and Lewd prefer P to L . So, by PARETO, $P \succ_s L$.

No Acceptable Choice: We shouldn't choose \emptyset because (from Lewd's LIBERTY) L is socially preferred to it. We shouldn't choose L because (from PARETO) P is socially preferred to it. And we shouldn't choose P because (from Prude's LIBERTY) \emptyset is socially preferred to it.

Lessons?

Should we give up PARETO, LIBERTY, UNRESTRICTED DOMAIN, or just live with the consequences?

- *Reject UNRESTRICTED DOMAIN*. We shouldn't worry about making sensible collective choices given *any* preference profile—some are too strange or outlandish or perverse to worry about!
Response: These don't seem like strange preferences to have. (Maybe, though, living together in a liberal way involves inculcating preferences that respect the liberty of others?)
- *Reject LIBERTY*. We shouldn't understand an individual's right to choose between two alternatives as the right to determine the relative ordering of these alternatives within a social ordering.
- *Reject PARETO*. Just because everyone prefers one outcome to another doesn't make it better for society—perhaps this is especially so when the preferences in question are "nosey" ones, regarding what others decide to do in their personal spheres of liberty.

Prude prefers no one to read the book. But, if someone has to, Prude would rather it be her—unlike Lewd, she thinks, her moral convictions are strong enough to resist the book's corrupting influence. So, her preferences are:

$$\emptyset \succ_p P \succ_p L \succ_p E$$

Lewd, on the other hand, prefers everyone to read the book. But, if only one person can read it, Lewd would rather it be Prude—she thinks it might help Prude to loosen up a bit, and so Prude would benefit more from reading it than Lewd would. So, her preferences are:

$$E \succ_l P \succ_l L \succ_l \emptyset$$

Notice that if we accept *Transitivity of (Strict) Social Preference*, it follows that: $L \succ_s P$.

And that straightforwardly contradicts the result of PARETO. (We needn't assume *Transitivity*, however, to generate the conflict.)

We also shouldn't choose E because (from Prude's LIBERTY) L is socially preferred to it.

Lesson: In a truly liberal society, Prude and Lewd wouldn't have preferences that give rise to the conflict—in particular, they wouldn't both prefer P to L .

Lesson: We should understand *liberty*, not as a constraint about which outcomes should be socially ranked ahead others, but about which issues should be delegated to individual decision-making. Allowing individuals to exercise their liberty needn't result in "socially better" outcomes!

Lesson: Maybe everyone wanting something is only relevant if they want that thing for the right reasons.