Game Theory

Thinking Strategically II: Nash Equilibria

Review:

Prisoners' Dilemma

A game in which each player has a dominant strategy, resulting in an outcome that is collectively worse than some other.

Dominance: Strict vs Weak

Strict: it's guaranteed to be better no matter what the other players do.

Weak: it's guaranteed to be *at least as good* or *better* no matter what the other players do.

Pareto-dominance

Everyone (i.e., each player) prefers that outcome to the other.

Review:

Thinking Strategically:

1. **Don't play dominated strategies**
   Why? *Because they are guaranteed to be worse than something else you could do.*

2. **Put Yourself in the Other's Shoes**
   Think about the game from the other player's perspective.
   If they have a dominated strategy, you can assume they won't play it.
   Eliminate it, and reassess the game.

Review:

Thinking Strategically:

1. **Don't play dominated strategies**
   Why? *Because they are guaranteed to be worse than something else you could do.* [What about *weak dominance*, though?]

2. **Put Yourself in the Other's Shoes**
   Think about the game from the other player's perspective.
   If they have a dominated strategy, you can assume they won't play it.
   Eliminate it, and reassess the game. [Relies on *common knowledge*.]

The Number Game

The Number Game

Choose a number between 1 and 100.

The number that is closest to two-thirds of the average wins the prize.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies.

The Number Game

How might you approach playing this game?

Suppose everyone picked 100.
Then, average = 100.

The Number Game

How might you approach playing this game?

Suppose everyone picked 100.
Then, average = 100.
So, Winning # = 66.66666

"100" is the highest the average could possibly be.

The Number Game

How might you approach playing this game?

Suppose everyone picked 100.
Then, average = 100.
So, Winning # = 66.66666
So, picking 67 weakly dominates all higher numbers.

"100" is the highest the average could possibly be.
So, all guesses higher than 67 are too high to be close to 2/3rds the average.

The Number Game

How might you approach playing this game?

Suppose everyone picked 100.
Then, average = 100.
So, Winning # = 66.66666
So, picking 67 weakly dominates all higher numbers.

Life Lesson:
Don't play dominated strategies

So...
Don't pick a number higher than 67.

The Number Game

How might you approach playing this game?

Suppose everyone picked 100.
Then, average = 100.
So, Winning # = 66.66666
So, picking 67 weakly dominates all higher numbers.

Life Lesson:
Don't play dominated strategies

So...
Don't pick a number higher than 67

Put Yourself in Other's Shoes:
Assume they won't play dominated strategies

So...
Assume that no one will pick a number higher than 67.

The Number Game

How might you approach playing this game?

Suppose everyone picked 67.
Then, average = 67.
So, Winning # = 44.66666
So, picking 45 weakly dominates the remaining strategies.

The Number Game

How might you approach playing this game?

Suppose everyone picked 45.
Then, average = 45.
So, Winning # = 30
So, picking 30 weakly dominates the remaining strategies.

The Number Game

How might you approach playing this game?

Suppose everyone picked 30.
Then, average = 30.
So, Winning # = 20
So, picking 20 weakly dominates the remaining strategies.

The Number Game

How might you approach playing this game?

Suppose everyone picked 20.
Then, average = 20.
So, Winning # = 13.3333
So, picking 13 weakly dominates the remaining strategies.

The Number Game

How might you approach playing this game?

… and so on and so forth ...

The Number Game

How might you approach playing this game?

… and so on and so forth …

… until we reach 1.

The Number Game

How might you approach playing this game?

… and so on and so forth …

… until we reach 1.

Rational Solution: guess 1?

The Number Game

Rational Solution: guess 1?

Not necessarily!

That argument made a strong assumption:
Common knowledge of rationality.

Common Knowledge

Common Knowledge

Some fact (call it "*p*") is *common knowledge* just in case (1) everyone knows that *p*, (2) everyone knows that everyone knows that *p*, (3) everyone knows that everyone knows that everyone knows that *p*, ...

The Number Game

If you (all) are rational, you won't guess numbers higher than 67.

But there might not be *common knowledge* of rationality.

Do you know that everyone knows that everyone knows that everyone knows that everyone knows that … everyone is rational?

The Number Game

In Game Theory, we assume *common knowledge of rationality*.

This is a strong assumption, which might not hold in reality. (So keep that in mind!)

The Number Game

In Game Theory, we assume *common knowledge of rationality*.

This is a strong assumption, which might not hold in reality. (So keep that in mind!)

Note: Asserting p (even when everyone already knows that p) is often a way to make p common knowledge.

The Number Game

Two Variations on the Number Game

Two Player Version:
Pick a number between 1-100.
You win if your number is closest to 2/3rds of the other player's number.

Which number should you choose?

The Number Game

Two Variations on the Number Game

Keynes' Beauty Contest Example:
Pick a number between 1-100.
You win if your number is closest to the average guess.

Which number should you choose?

THE SIX WINNERS in the BEAUTY CONTESTS of THE SUNDAY TIMES

The Number Game

Two Variations on the Number Game

Keynes' Beauty Contest Example:
Pick a number between 1-100.
You win if your number is closest to the average guess.

Keynes thought this is sort of like a stock market bubble.

THE SIX WINNERS in the BEAUTY CONTESTS of THE SUNDAY TIMES

Nash Equilibria

Nash Equilibria

Set of strategies, one for each player, such that no player has an incentive to change their strategy.

Nash Equilibria

Rock, Paper, Scissors

Rock, Paper, Scissors

W = win
L = lose
0 = tie

		2		
		r	**p**	**s**
	R	$0,0$	L,W	W,L
1	**P**	W,L	$0,0$	L,W
	S	L,W	W,L	$0,0$

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

		2		
		r	**p**	**s**
	R	$0,0$	$-1,1$	$1,-1$
1	**P**	$1,-1$	$0,0$	$-1,1$
	S	$-1,1$	$1,-1$	$0,0$

(Note: This is a zero-sum game.)

Best Response

Best Response

Given what all other players are doing, a strategy is a *best response* just in case a player cannot do better by switching to a different strategy.

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

			2	
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 2 plays **r**ock, Player 1 should…

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

			2	
		r	p	s
	R	0,0	−1,1	1,−1
1 →	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 2 plays **r**ock, Player 1 should choose **P**aper.

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

			2	
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 1 plays **P**aper, Player 2 should…

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

			2	↓
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 1 plays **P**aper, Player 2 should choose **s**cissors.

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

2

		r	p	s
	R	0, 0	−1, 1	1, −1
1	P	1, −1	0, 0	−1, 1
	S	−1, 1	1, −1	0, 0

If Player 2 plays **s**cissors, Player 1 should...

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

2

		r	p	s
	→ R	0, 0	−1, 1	1, −1
1	P	1, −1	0, 0	−1, 1
	S	−1, 1	1, −1	0, 0

If Player 2 plays **s**cissors, Player 1 should choose **R**ock.

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

2

		r	p	s
	R	0, 0	−1, 1	1, −1
1	P	1, −1	0, 0	−1, 1
	S	−1, 1	1, −1	0, 0

If Player 1 plays **R**ock, Player 2 should...

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

2
↓

		r	p	s
	R	0, 0	−1, 1	1, −1
1	P	1, −1	0, 0	−1, 1
	S	−1, 1	1, −1	0, 0

If Player 1 plays **R**ock, Player 2 should choose **p**aper.

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

			2	
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 2 plays **p**aper, Player 1 should...

Rock, Paper, Scissors

1 = win
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			2	
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	→S	−1,1	1,−1	0,0

If Player 2 plays **p**aper, Player 1 should choose **S**cissors.

Rock, Paper, Scissors

1 = win
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			2	
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 1 plays **S**cissors, Player 2 should...

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

			2	
		↓ r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 1 plays **S**cissors, Player 2 should choose **r**ock.

Rock, Paper, Scissors

1 = win
-1 = lose
0 = tie

		2		
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

If Player 2 plays rock, Player 1 should...

Rock, Paper, Scissors

In “Rock, Paper, Scissors”, there is no *stable* set of strategies.

		2		
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

Rock, Paper, Scissors

In “Rock, Paper, Scissors”, there is no *stable* set of strategies.

		2		
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

Rock, Paper, Scissors

In “Rock, Paper, Scissors”, there is no *stable* set of strategies.

The best response to Rock is Paper,
The best response to Paper is Scissors,
The best response to Scissors is Rock, …
… and so on and so forth …

		2		
		r	p	s
	R	0,0	−1,1	1,−1
1	P	1,−1	0,0	−1,1
	S	−1,1	1,−1	0,0

Best Response

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Given what all other players are doing, a strategy is a *best response* just in case a player cannot do better by switching to a different strategy.

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Given what all other players are doing, a strategy is a *best response* just in case a player cannot do better by switching to a different strategy.

Example: The best response to Rock is Paper; the best response to Paper is Scissors; the best response to Scissors is Rock

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Best Response

If you knew what the other player would do, you should play your best response to their move.

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Likewise, if the other player knew what you would do, they should play their best response to your move.

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In order to know what to do, it’s helpful to know what the other player will do.

Best Response

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Likewise, if the other player knew what you would do, they should play their best response to your move.

In order to know what to do, it’s helpful to know what the other player will do.

Best Response

If you knew what the other player would do, you should play your best response to their move.

Likewise, if the other player knew what you would do, **they should play their best response to your move.**

In order to know what to do, it’s helpful to know what the other player will do.

What the other player will do depends on what they think *you* might do.

Best Response

If you knew what the other player would do, you should play your best response to their move.

Likewise, if the other player knew what you would do, they should play their best response to your move.

You are trying to predict what *they* will do.

They are trying to predict what *you* will do.

Best Response

Sometimes (like in "Rock, Paper, Scissors"), there is no *stable* stopping point.

Best Response

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Example:

Best Response

Sometimes (like in "Rock, Paper, Scissors"), there is no *stable* stopping point.

Example:

		2	
		Poison **A**	Posion **B**
1	Drink **A**	Die, Live	Live, Die
	Drink **B**	Live, Die	Die, Live

Best Response

Sometimes (like in "Rock, Paper, Scissors"), there is no *stable* stopping point.

Example:

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Example:

2

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Best Response

Sometimes (like in "Rock, Paper, Scissors"), there is no *stable* stopping point.

Example:

2

		Poison **A**	Posion **B**	
1	Drink **A**	Die, Live	Live, Die	
	Drink **B**	Live, Die	Die, Live	

Best Response

Sometimes (like in "Rock, Paper, Scissors"), there is no *stable* stopping point.

Example:

2

		Poison **A**	Posion **B**	Poison **Both**
1	Drink **A**	Die, Live	Live, Die	
	Drink **B**	Live, Die	Die, Live	

Best Response

Sometimes (like in “Rock, Paper, Scissors”), there is no *stable* stopping point.

Example:

And develop an immunity to iocane powder

		2		
		Poison **A**	Posion **B**	Poison **Both**
1	Drink **A**	Die, Live	Live, Die	
	Drink **B**	Live, Die	Die, Live	

Best Response

Sometimes (like in “Rock, Paper, Scissors”), there is no *stable* stopping point.

Example:

And develop an immunity to iocane powder

		2		
		Poison **A**	Posion **B**	Poison **Both**
1	Drink **A**	Die, Live	Live, Die	Die, Live
	Drink **B**	Live, Die	Die, Live	Die, Live

Best Response

Sometimes (like in “Rock, Paper, Scissors”), there is no *stable* stopping point.

Example:

And develop an immunity to iocane powder

		2		
		Poison **A**	Posion **B**	Poison **Both**
1	Drink **A**	Die, Live	Live, Die	Die, Live
	Drink **B**	Live, Die	Die, Live	Die, Live

Nash Equilibria

Nash Equilibria

Example: What should you do if you think Player 2 will play **l**?

		2	
		l	**r**
1	**U**	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

Example: What should you do if you think Player 2 will play **l**?

		2	
		l	**r**
1	**U**	2, 1	1, 2
	→ **D**	4, 5	0, 10

Nash Equilibria

Example: What should Player 2 do if you play **D**?

		2	
		l	**r**
1	**U**	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

Example: What should Player 2 do if you play **D**?

		2	↓
		l	**r**
1	**U**	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

Example: What should you do if Player 2 plays **r**?

2

		l	**r**
1	**U**	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

Example: What should you do if Player 2 plays **r**?

2

		l	**r**
1	→**U**	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

Example: What should Player 2 do if you play **U**?

2

		l	**r**
1	**U**	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

Example: What should Player 2 do if you play **D**?

2

		l	↓ **r**
1	**U**	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

Example: If you play **U**, Player 2 should play **r**.
And, if Player 2 plays **r**, you should play **U**.

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{2} \\ & & \mathbf{l} & \mathbf{r} \\ \hline 1 & \mathbf{U} & 2,1 & \boxed{1,2} \\ \hline & \mathbf{D} & 4,5 & 0,10 \\ \hline \end{array}$$

Nash Equilibria

Set of strategies, one for each player, such that no player has an incentive to change their strategy.

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Set of strategies, one for each player, such that no player has an incentive to change their strategy.

No Regrets: no player can do strictly better by deviating (holding fixed what everyone else does).

Nash Equilibria

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No Regrets: no player can do strictly better by deviating (holding fixed what everyone else does).

Self-fulfilling beliefs: If you predict that everyone will play their part of a NE, everyone will.

Nash Equilibria

WHY?

Self-fulfilling beliefs: If you predict that everyone will play their part of a NE, everyone will.

Activity: Investment Game

Nash Equilibria

Example: Investment Game

Players:	you	
Strategies:	invest $0 or invest $10	
Payoffs:	if you invest $0,	you win/lose nothing
	if you invest $10,	win $11 if >90% invests win $0 otherwise.

Nash Equilibria

Example: Investment Game

		2	
		invest	refrain
1	Invest	1, 1	−10, 0
	Refrain	0, −10	0, 0

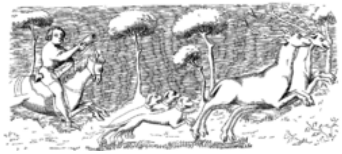
Nash Equilibria

Example: Bank Run

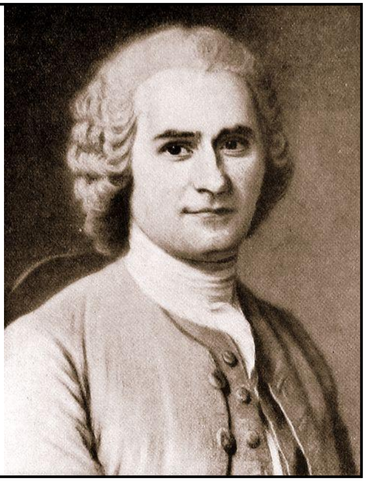
Example: Stag Hunt

Nash Equilibria

Example: Stag Hunt



Jean Jacques Rousseau (1712-1778)
The Discourse on the Origin of Inequality



Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1

Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3, 3	0, 2
	Hare	2, 0	1, 1

Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	→ Stag	3, 3	0, 2
	Hare	2, 0	1, 1

Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3, 3	0, 2
	Hare	2, 0	1, 1

Nash Equilibria

Example: Stag Hunt

		2	
		↓ stag	hare
1	Stag	3, 3	0, 2
	Hare	2, 0	1, 1

Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1

Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3,3	0,2
	→ Hare	2,0	1,1

Nash Equilibria

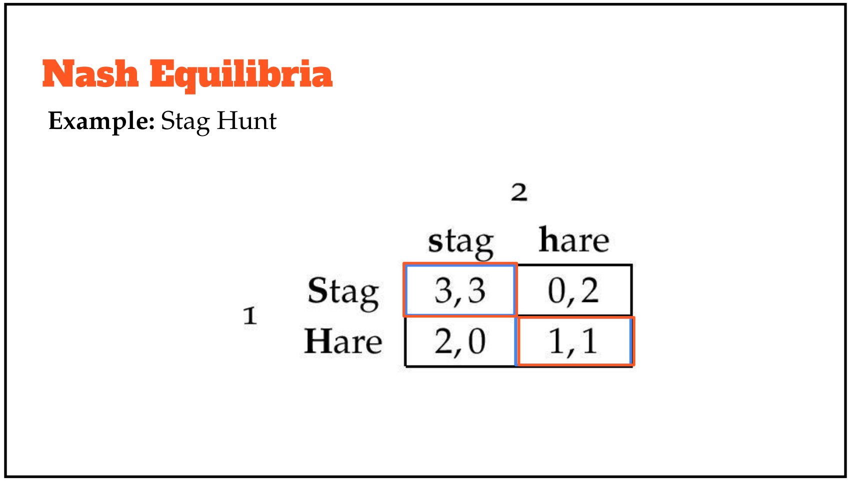
Example: Stag Hunt

		2	
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1

Nash Equilibria

Example: Stag Hunt

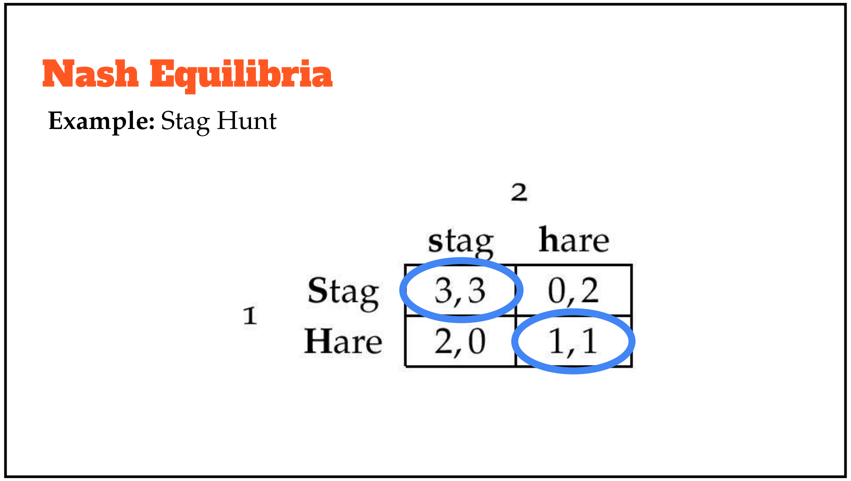
		2	↓
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1



Nash Equilibria

Example: Stag Hunt

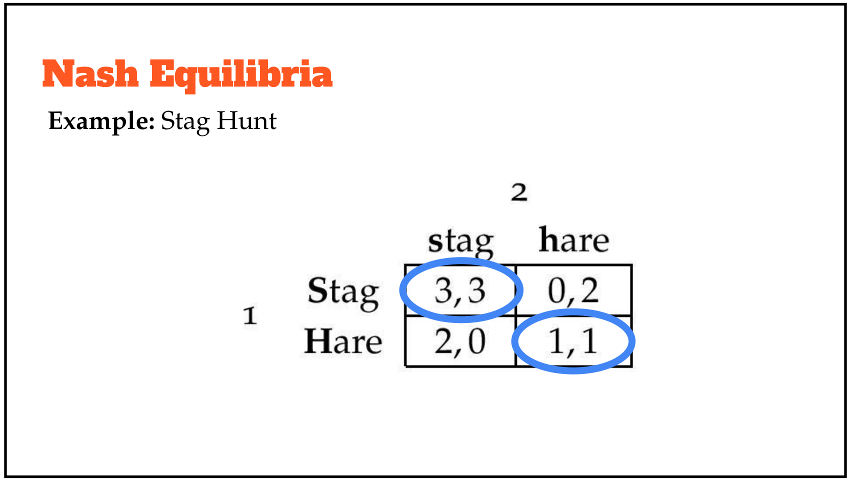
		2	
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1



Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1



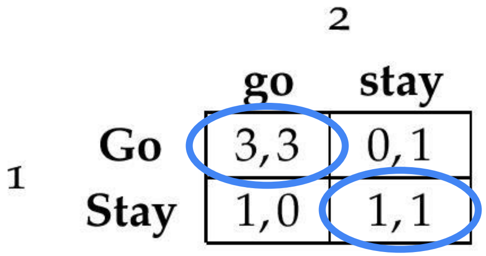
Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1

Example: Meeting Game

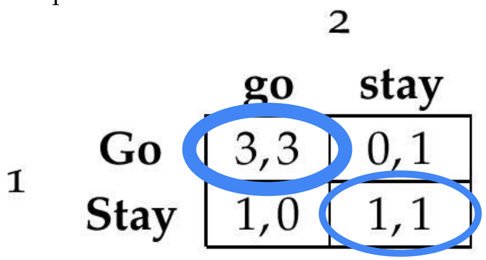
Meeting Game (Stag Hunt)



		2	
		go	stay
1	Go	3,3	0,1
	Stay	1,0	1,1

Meeting Game (Stag Hunt)

Communication helps!



		2	
		go	stay
1	Go	3,3	0,1
	Stay	1,0	1,1

Example: Stoplight

Nash Equilibria

Example: Stoplight game

		2	
		go	stop
1	Go	–5, –5	1, 0
	Stop	0, 1	–1, –1

Nash Equilibria

Example: Stoplight game

		2	
		go	stop
1	Go	−5, −5	1,0
	Stop	0, 1	−1, −1

Example: The Prisoners' Dilemma

Nash Equilibria

		Them	
		A	**B**
You	**A**	0,0	2, −1
	B	−1,2	1, 1

Nash Equilibria

		Them	
		A	**B**
You	**A**	0,0	2, −1
	B	−1,2	1, 1