

# Counterexamples to Transitivity

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## An Argument for Transitivity

As we saw when we looked at the Representation Theorems, a crucial rational constraint on your preferences is that they be transitive:

**Transitivity of Preference** If  $X \succ Y$  and  $Y \succ Z$ , then  $X \succ Z$ .

Are you irrational if your preferences aren't transitive? Here's a famous argument that you are: *The Money Pump Argument*.

### MONEYPUMP ARGUMENT

<p><b>P1</b> If you're rational, you should act on the basis of your rational preferences (e.g., if <math>X \succ Y</math>, then there is some small amount <math>\\$ \epsilon</math> such that you ought to be willing to pay <math>\\$ \epsilon</math> to trade <math>Y</math> for <math>X</math>).</p> <p><b>P2</b> If <math>A \succ B \succ C \succ A</math>, and you act on your preferences, then you will act as a money pump.</p> <p><b>P3</b> It's not rational to act as a money pump.</p> <hr/> <p><b>C</b> It's not rational to have the preferences: <math>A \succ B \succ C \succ A</math>.</p>
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Here's the idea. Suppose you have  $A$ . Because you prefer  $C$  to  $A$ , you should be willing to pay  $\$ \epsilon$  to trade  $A$  for  $C$ . Because you prefer  $B$  to  $C$ , you should be willing to pay  $\$ \epsilon$  to trade  $C$  for  $B$ . Because you prefer  $A$  to  $B$ , you should be willing to pay  $\$ \epsilon$  to trade  $B$  for  $A$ . You started with  $A$  and have been led to make a series of trades that have left you strictly worse off.

## Intransitivity & Individuating Alternatives

Here's an example of preferences that, at first glance, appear to violate Transitivity:

Maurice, given a choice between going mountaineering in the Alps and visiting Rome, prefers to visit Rome. Given a choice between staying at home and visiting Rome, he prefers to stay at home. But given a choice between staying at home and going mountaineering, he prefers to go mountaineering.

Broome points out that whether Maurice's preferences are intransitive depends on how we individuate the alternatives.

$H_r$ : Maurice stays home, when going to Rome was the only available alternative.

If your preferences *aren't* transitive, then they cannot be represented with a utility-function at all.

Donald Davidson, J. McKinsey, and Patrick Suppes, "Outlines of a Formal Theory of Value," *Philosophy of Science*, 22. 1955.

MONEY PUMP

Round 0:	A
Round 1:	C - $\$ \epsilon$
Round 2:	B - $\$ 2 \epsilon$
Round 3:	A - $\$ 3 \epsilon$
⋮	⋮

The example is from John Broome, "Can a Humean be Moderate?" In *Ethics out of Economics*, Cambridge University Press. 1999.

M: Maurice goes mountaineering.

R: Maurice goes to Rome.

H: Maurice stays home.

$M \succ H \succ R \succ M$ .

M: Maurice goes mountaineering.

R: Maurice goes to Rome.

$H_m$ : Maurice stays home, when mountaineering was the only available alternative.

Can all (alleged) counterexamples to Transitivity be handled in this way (by individuating the alternatives more finely)?

### The Continuum Counterexample to Transitivity

Temkin’s counterexample to the transitivity of “better than” relies on three claims:

- Claim 1:** for any unpleasant experience, no matter its intensity or duration, it would be better to have that experience than one that was only slightly less intense but twice as long.
- Claim 2:** there is a continuum of unpleasant experiences ranging in intensity from extreme agony to very mild discomfort.
- Claim 3:** very mild discomfort, no matter how long it must be endured, it better than suffering extreme agony for a significant amount of time.

FROM TORTURE TO HANGNAILS			
Case 0:	pain of intensity 100	for	1 day
Case 1:	pain of intensity 99	for	2 days
Case 2:	pain of intensity 98	for	3 days
⋮	⋮		⋮
Case $n$ :	pain of intensity $100 - n$	for	$2^n$ days
⋮	⋮		⋮
Case 99:	pain of intensity 1	for	$2^{99}$ days

Broome worries that such a move would make Transitivity trivial unless a justification can be given for placing  $H_r$  and  $H_m$  in different places in Maurice’s preference ordering.

For example, “[s]uppose the explanation of Maurice’s preferences is this. He is frightened of heights, and therefore would rather go to Rome than go mountaineering. Sightseeing bores him, however, and therefore he would rather stay at home than go to Rome. But Maurice sees a choice of staying at home and undertaking a mountaineering trip as a test of his courage. He believes it would be cowardly to stay home, and that is why he prefers to go mountaineering. (He considers it cultured, not cowardly, to visit Rome.)”

Assume that pain of intensity 100 is extreme agony (e.g., torture), and that pain of intensity of 1 is very mild discomfort (e.g., a hangnail).

For each  $k$ , Case  $k$  is better than Case  $k + 1$ . But Case 99 is better than Case 0. Therefore, “better than” is not transitive.

### Responses to the Argument?

1. **Deny Claim 3.** Our intuitions concerning very large numbers — like the duration that the very mild discomfort must be endured — are unreliable. Pain of intensity 1 for  $2^{99}$  days really is worse than pain of intensity 100 for 1 day. [Broome]
2. **Deny the Argument’s Validity.** Consider an agent that maximizes the following utility-function:  $u(i, d) = -\frac{i \cdot d}{1 + d}$ , where  $i$  is the intensity of pain and  $d$  is its duration. [Binmore & Voorhoeve]
3. **Deny Claim 1.** All else equal, more intense pains are worse than less intense pains; all else equal, pains of longer duration are worse than pains of shorter duration. But the these two dimensions of badness aren’t precisely commensurable. So there are adjacent cases such that neither is worse than the other, nor are they equally bad. [Handfield]